Author

Caitlin Sikora felt that a research experience would be valuable in deciding on a career path or future area of study so she decided to complete a senior thesis to finish her physics major. She chose to work with Dr. Chen, who offered her a rare opportunity to work on particle theory as an undergraduate, in a very specific and exciting branch of particle physics. In addition to her major in physics, Caitlin has completed a major in dance, dancing and choreographing in many UCI Dance Department performances. Over the next few years she hopes to pursue a dance career before moving on to graduate school in physics.

Key Terms

- Antimatter
- CP-Symmetry
- Leptogenesis
- Neutrino
- Neutrino Flavor Oscillations

Physical Implications of Neutrino Mixing Angles and CP-Violating Phase

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Abstract

The origin of the asymmetry between matter and antimatter in the universe remains a great mystery. In 1998, neutrino oscillations were observed in atmospheric neutrinos, shattering assumptions that neutrinos were massless and suggesting a possible violation of change and parity symmetry (CP-symmetry) in the neutrino sector. This suggests that leptogenesis is possible, hypothetically generating leptons in greater quantities than antileptons, potentially explaining the asymmetry between matter and antimatter, which makes existence possible. By deriving the neutrino mixing matrix and expanding it in terms of small deviations about the Tri-bimaximal mixing pattern to the third order, it has been shown that slight variations in these parameters can significantly affect flavor transition probabilities and the possibility of leptogenesis. I examined the dependence of each transition probability on each mixing angle, Dirac CP-violating phase, and mass ordering, identifying the electron to muon and tau to electron flavor channels as the most sensitive to such variations. I also calculated leptogenesis in terms of the Dirac CP-violating phase for a model derived from the double tetrahedral T' group theory, finding it to be nonzero with flavor effects included. These results may guide future experiments and model building in hopes of observing CP-violation and explaining the matter-antimatter asymmetry.

Faculty Mentor

The recent discovery of neutrino oscillation—that is, that neutrinos morph from one type to another during their free flight in space—has given solid evidence of the existence of tiny but non-zero neutrino masses. Massive neutrinos may also explain the asymmetry between matter and antimatter in the Universe. Sikora has investigated the phenomenological implications for neutrino oscillation and the matter-antimatter asymmetry in a theoretical model based on the symmetry that also describes the structure of methane. This highly predictive model unifies three of the four fundamental forces in nature into a single grand unified interaction. The model's predictions for the mixing angles will be tested in upcoming neutrino oscillation experiments.

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Introduction

The Two Great Mysteries of Particle Physics

An antiparticle is a constituent of antimatter, just as a particle is a constituent of matter; it possesses nearly all the same properties as its associated particle, but with opposite charge. For example, a positron, the antiparticle of an electron, has the same mass and spin as an electron, but with positive rather than negative charge (Anderson, 1933). As we may have imagined, it is true that if the quantities of matter and antimatter in the known universe were equal, the two would indeed annihilate each other, thus terminating existence as we know it (Sather, 1996); fortunately, this is not the case. The observed quantities of antiparticles are far smaller than those of particles, resulting in a huge asymmetry between matter and antimatter, which has yet to be fully accounted for by the theories of modern physics.

To explain the asymmetry between matter and antimatter, the violation of charge and parity symmetry, or CP-symmetry must be observed. CP-symmetry exists if the laws of physics are preserved under changes in both charge and parity: if a particle is exchanged with its antiparticle, giving both opposite charge, and they are reflected over space, giving both opposite parity, their behavior should not change (Sather, 1996). CP-violation has been observed in the quark sector, in which the process called baryogenesis gives rise to the asymmetry between a subset of particles called baryons and antibaryons, but not in magnitudes large enough to account for the total observed asymmetry between matter and antimatter (Sather, 1996). Where, then, is the remaining antimatter? Where in the formation of the universe did this remaining asymmetry arise?

Another great mystery in particle physics is the problem of missing neutrinos at the earth's surface. Neutrinos are tiny, electrically neutral particles that were once thought to be massless. Since their discovery in 1956, thanks in part to UCI's Frederick Reines, three neutrino flavors—electron, muon and tau—have been observed, each created in reactions with their corresponding leptons, another subset of particles (Freedman, 2004). Nuclear reactions in the sun, for example, produce electron-flavor neutrinos, and their flux can be predicted by the Standard Solar Model. In the late 1960s, however, the flux of electron neutrinos at earth's surface was measured to be around 1/3 of the predicted value, a phenomenon known as the solar neutrino problem (Freedman, 2004). How do we account for the missing electron neutrinos? If not at the earth's surface, where did the remaining electron neutrinos created by the sun's reactions end up?

Neutrino flavor oscillations, a quantum mechanical effect first suggested in 1957 by Bruno Pontecorvo, provide a solution. According to quantum mechanics, a neutrino of a given flavor can be described by its wave function. Each flavor wave function is a linear combination of three mass eigenstate wave functions, which evolve according to given mass-specific energy parameters. Since the mass eigenstate wave functions evolve differently over time, there are often nonzero probabilities of observing neutrinos of changed flavor (Freedman, 2004). Neutrino flavor oscillations resolve the solar neutrino problem, postulating that electron neutrinos produced by the sun change flavor during their journey to earth. These flavor oscillations, however, also require at least two unique nonzero mass eigenvalues for neutrinos, thus contradicting the Standard Model of particle physics, which predicts massless neutrinos (Gonzales-Garcia, 2003). This implication opens the possibility of CP-violation in the neutrino sector, which supports the prospect of leptogenesis, a process that gives rise to the asymmetry between leptons and antileptons through the decay of the right-handed neutrinos. Specifically, if CP-symmetry is violated, the decay rates of a right-handed neutrino into a lepton and into an antilepton will be different, thus generating a matter asymmetry. The see-saw mechanism that has been suggested to generate the observed neutrino masses, which makes leptogenesis possible, meets the necessary conditions for explaining baryon asymmetry. This could account for the remaining asymmetry between matter and antimatter (Nir, 2007). Through the see-saw mechanism, it is also possible to relate neutrino oscillation parameters—low energy physical observables—to leptogenesis, which takes place at scales 10^{12} orders of magnitude and is not accessible experimentally.

In 1998, neutrino flavor changes were experimentally verified in atmospheric neutrinos, providing solid evidence for nonzero neutrino masses and opening up the possibility of leptogenesis (Super-Kamiokande, 1998). Although the absolute values of these neutrino masses are extremely small and difficult to measure, the differences between these mass eigenvalues, as well as two of the three neutrino mixing angles in the proposed mixing matrix, which governs neutrino flavor oscillations, have been narrowed down to specific ranges by experimental and theoretical work. The current best-fit model of the neutrino mixing matrix, the so-called Tri-bimaximal mixing matrix (Harrison, 2002), does not incorporate CP-violation, which is needed for leptogenesis; however, the uncertainty in the two parameters that determine CP-violation is still considerable. If small deviations in the CP-violating parameters from the best-fit model exist, leptogenesis, which generates matter-antimatter asymmetry using the decays of right-handed neutrinos, is still a possibility.

This project explores the prospect of leptogenesis as a means of explaining the observed asymmetry between matter and antimatter by considering deviations from the Tri-bimaximal values for the neutrino mixing angles. First, I examine the effects of deviations from the best-fit neutrino mixing parameters on the probabilities of neutrinos transitioning from one flavor to another. By exploring all nine possible flavor transitions, I identify the channels most sensitive to CP-violation, as well as the ranges for the distance to energy ratio in which the effects of CP-violation and the choice of the mass hierarchy model are most prominent. I then move on to investigate the possibility of leptogenesis in a particular model derived from group theory, mapping the deviations from the Tri-bimaximal mixing parameters to the values predicted by the model.

The results may guide the development of models for explaining neutrino masses, the efforts of future experiments in the search for CP-violation in the neutrino sector and the quest to improve the accuracy of neutrino masses and mixing angles.

Deviations from Tri-bimaximal Mixing and Transition Probabilities

The following section sets up the structure of neutrino oscillations, introducing the neutrino mixing matrix and angles. I then expand the matrix about deviations from the Tri-bimaximal values for the neutrino mixing angles, moving on to calculate flavor transition probabilities in terms of these small deviations. Using plots of these transition probabilities, I explore the channels most sensitive to CP-violation and changes in the mass ordering. Directing future neutrino experiments toward these sensitive channels may increase the likelihood of observing CP-violation in the neutrino sector or improving the accuracy of neutrino masses and mixing parameters.

The Neutrino Mixing Matrix (MNS) and Mixing Angles

The neutrino mixing matrix governs the evolution of the state of a neutrino. This unitary matrix can be parameterized in the following way (Yao, 2006):

$$U(\theta_{23},\theta_{13},\theta_{12},\delta) = R_{23}(\theta_{23})U_\delta^\dagger R_{13}(\theta_{13})U_\delta R_{12}(\theta_{12})$$

$$U(\theta_{23},\theta_{13},\theta_{12},\delta) = \begin{bmatrix} \cos\theta_{12}\cos\theta_{13} & \sin\theta_{12}\cos\theta_{13} & \sin\theta_{13}e^{-i\delta} \\ -\sin\theta_{12}\cos\theta_{23} - \cos\theta_{12}\sin\theta_{23}\sin\theta_{13}e^{i\delta} & \cos\theta_{12}\cos\theta_{23} - \sin\theta_{12}\sin\theta_{23}\sin\theta_{13}e^{i\delta} & \sin\theta_{23}\cos\theta_{13} \\ \sin\theta_{12}\sin\theta_{23} - \cos\theta_{12}\cos\theta_{23}\sin\theta_{13}e^{i\delta} & -\cos\theta_{12}\sin\theta_{23} - \sin\theta_{12}\cos\theta_{23}\sin\theta_{13}e^{i\delta} & \cos\theta_{23}\cos\theta_{13} \end{bmatrix} \quad (1)$$

In the derivation of the mixing matrix above, R_{jk} indicates a rotation in the jk-plane of the Hilbert space through angle θ_{jk}. The matrix U_δ is defined as the diagonal matrix of elements diag($e^{i\delta/2}$, 1, $e^{-i\delta/2}$), where δ is the CP-violating phase, which determines the magnitude in which CP-symmetry is violated. In the above parameterization, the complex Majorana phases have been omitted as they do not affect neutrino oscillations.

The experimental best-fit values and 3-sigma allowed range (numbers quoted in the parentheses) for the mixing angles are (Super-Kamiokande, 1998):

$\sin^2\theta_{12} = 0.30\ (0.25–0.34)$
$\sin^2\theta_{23} = 0.5\ (0.38–0.64)$
$\sin^2\theta_{13} = 0\ (< 0.028)$

The angle θ_{12} is the solar mixing angle measured through solar neutrino experiments such as KamLAND and SNO. Tthe angle θ_{23} is the atmospheric mixing angle measured through atmospheric neutrino experiments like Super-K. Measuring the angle θ_{13} is the major goal of most current and planned upcoming experiments. These values agree very well with the proposed Tri-bimaximal mixing matrix (Harrison, 2002):

$$U_{TBM} = R_{23}(\pi/4)\, R_{12}(\sin^{-1}(1/\sqrt{3})) = \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} \quad (2)$$

This predicts $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$, $\theta_{13} = 0$, and $\delta = 0$, and was verified by setting the Tri-bimaximal mixing matrix equal to the general mixing matrix and comparing corresponding elements to solve for the values of each θ and δ. Because this method yields both θ_{13} and δ equal to zero, the Tri-bimaximal matrix implies CP-invariance in the neutrino sector, but CP-violation is required to generate the observed asymmetry between matter and anti-matter in the universe. This asymmetry is consistent with the bounds placed on the relative quantities of baryons and anti-baryons for baryogenesis. Although bounds have not been placed on the relative numbers of leptons and antileptons, leptogenesis, which requires CP-violation in the neutrino sector, looks promising. We are thus left with two options to account for CP-violation: deviations from the Tri-bimaximal mixing parameters and the introduction of the Majorana phases. The remainder of this paper discusses the former.

Expansion of the Matrix

We explore the allowed range of values for the mixing angles and CP-violating phase, as Pakvasa, Rodejohann and Weiler suggest (Pakvasa, 2008), by parameterizing the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix in terms of small deviations from the Tri-bimaximal values for θ_{13}, θ_{12} and θ_{23}, as well as δ. The resulting matrix can be expanded about these deviations and the phase δ up to higher orders, yielding a matrix known as the Tri-minimal mixing matrix, which reduces to the Tri-bimaximal matrix on the zeroth-order. We accomplish this by naming small deviations from the Tri-bimaximal parameters ε_{13}, ε_{12}, ε_{23}, and δ, and then multiplying U_{TBM} by U (ε_{13}, ε_{12}, ε_{23}, δ), which rotates our neutrino states slightly farther about our axes. From this point, each sine and cosine term is Taylor expanded to third order, resulting in a matrix with elements in the form of products of polynomials. After each product is found, we collect terms of the same order and neglect those of order greater than three. This process is tedious, so I wrote a code in Mathematica, yielding the expanded form of the Tri-minimal parameterization of the mixing matrix U_{TMin}, which has been useful in probability calculations.

$$U_{TMin} = R_{23}(\pi/4)\, U(\varepsilon_{13}, \varepsilon_{12}, \varepsilon_{23}, \delta)\, R_{12}(\sin^{-1}(1/\sqrt{3}))$$

$$\begin{aligned}
U_{TMin} = U_{TBM} &- \frac{\varepsilon_{12}}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & -2 & 0 \\ \sqrt{2} & 1 & 0 \\ -\sqrt{2} & -1 & 0 \end{bmatrix} - \frac{\varepsilon_{23}}{\sqrt{6}}\begin{bmatrix} 0 & 0 & 0 \\ -1 & \sqrt{2} & -\sqrt{3} \\ -1 & \sqrt{2} & \sqrt{3} \end{bmatrix} - \frac{\varepsilon_{13}}{\sqrt{6}}\begin{bmatrix} 0 & 0 & -\sqrt{6}e^{-i\delta} \\ \sqrt{2}e^{i\delta} & e^{i\delta} & 0 \\ \sqrt{2}e^{i\delta} & e^{i\delta} & 0 \end{bmatrix} \\
&- \frac{\varepsilon_{12}^{2}}{2\sqrt{6}}\begin{bmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & 0 \\ 1 & -\sqrt{2} & 0 \end{bmatrix} - \frac{\varepsilon_{23}^{2}}{2\sqrt{6}}\begin{bmatrix} 0 & 0 & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{bmatrix} - \frac{\varepsilon_{13}^{2}}{2\sqrt{6}}\begin{bmatrix} 2 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} \end{bmatrix} \\
&- \frac{\varepsilon_{12}\varepsilon_{23}}{\sqrt{6}}\begin{bmatrix} 0 & 0 & 0 \\ -\sqrt{2} & -1 & 0 \\ -\sqrt{2} & -1 & 0 \end{bmatrix} - \frac{\varepsilon_{12}\varepsilon_{13}e^{i\delta}}{\sqrt{6}}\begin{bmatrix} 0 & 0 & 0 \\ -1 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & 0 \end{bmatrix} - \frac{\varepsilon_{13}\varepsilon_{23}e^{i\delta}}{\sqrt{6}}\begin{bmatrix} 0 & 0 & 0 \\ \sqrt{2} & 1 & 0 \\ -\sqrt{2} & -1 & 0 \end{bmatrix} + \ldots
\end{aligned} \tag{3}$$

Probability Calculations

With this expanded matrix, I have written a Mathematica code to calculate transition probabilities considering the full range of each parameter using the following formula, derived using the methods demonstrated by Kayser (2004). Here, L gives the distance traveled by the neutrinos, E gives the energy with which the neutrinos travel, and $\Delta m_{ij}^2 = m_i^2 - m_j^2$.

$$\begin{aligned}
P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) &= (\beta|\alpha(t)) \\
P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) &= \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}[U_{\alpha i}{}^* U_{\beta i} U_{\alpha j} U_{\beta j}{}^*]\sin^2(\Delta m_{ij}^2 \frac{L}{4E}) \pm 2\sum_{i>j} \mathrm{Im}[U_{\alpha i}{}^* U_{\beta i} U_{\alpha j} U_{\beta j}{}^*]\sin(\Delta m_{ij}^2 \frac{L}{2E})
\end{aligned} \tag{4}$$

Equation 4 shows the first instance of the dependence on neutrino masses in our calculations. The experimental values and 3-sigma allowed range (quoted in the parentheses) for the mass differences are given by (Pakvasa, 2008):

$\Delta m_{21}^2 = 8.1\ (7.5\text{–}8.7)\text{x}10^{-5}\ \text{eV}^2,$
$\Delta m_{32}^2 = 2.2\ (1.6\text{–}2.4)\text{x}10^{-3}\ \text{eV}^2,$
$\Delta m_{21}^2 + \Delta m_{32}^2 = \Delta m_{31}^2$

Only the relative splittings between m_2 and m_1 and between m_3 and m_2 are determined experimentally as evidenced in the above equations for the transition probabilities; the ordering of these masses is not determined except that $m_2 > m_1$. Figure 1 shows the two possible mass hierarchies, in which the flavor composition of each mass eigenstate is shown.

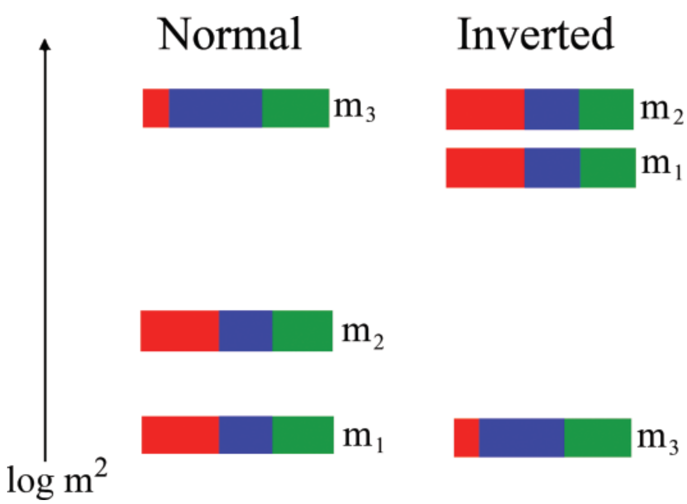

Figure 1

Normal (Left) and Inverted (Right) Neutrino Mass Hierarchies; Red indicates an electron neutrino state, blue a muon neutrino state, and green a tau neutrino state.

Each mass hierarchy is equally possible, but we will observe slight discrepancies in the predictions given by the different mass hierarchies, which arise only when we use a nonzero CP-violating phase δ. This δ-stipulation is obvious from the formula because $\sin^2(\theta) = \sin^2(-\theta)$, so the imaginary term must be nonzero, which requires a nonzero δ, in order for mass ordering to affect the probability. The other requirement for observing these discrepancies is that all three of the deviations from the Tri-bimaximal mixing matrix be nonzero, though it is not necessary for them to be at their maximum values.

Using Equation 4, I have written codes to calculate the probabilities of each of the nine possible flavor transitions in neutrinos, followed by those of antineutrinos, in terms of distance, energy, CP-violating phase, deviations from the Tri-bimaximal mixing matrix, and the mass splittings. With this code, substitution rules may be easily written and used to plot various scenarios or to find the dependence of probabilities on individual parameters.

Channels Most Sensitive to CP-Violation

One of the most powerful uses for this probability code is identifying the channels most sensitive to CP-violation, which may serve a vital role in the efforts to explain the observed matter-antimatter asymmetry. Using substitutions of values in the range allowed by experimental data, we can identify the dominant terms in each probability and maximize desired contributions. For example, we look at the probability of transitioning from muon flavor to muon flavor (the survival probability) to determine which term depends most strongly on ε_{13}, which gives the coefficient for the terms depending on δ. One way to find this term is to graph each of the three terms found in the probability calculation over ε_{13} to find the term with the highest probability for the allowed range of ε_{13}.

Now we find the distance (in km) to energy (in GeV) ratio L/E to maximize the dominant term, maximizing the effects of CP-violation on the transition probability, and we find the transition probability in terms of ε13. A similar procedure may be used for all of the transition probabilities. The value L/E may be used in designing experiments so that the effects of CP-violation will be maximized and digression from predictions of a given model will be most easily observable. For the case of mu to mu transitions, we fix E at 1 GeV and find L = 61,840 km to maximize the effects of CP-violation, giving an L/E value of 61,840 km/GeV.

Using this L/E value and maximum and minimum values for ε_{13}, we may plot each transition probability over the possible range of δ, from 0 to 2π. Note that in this method, the dependence of neutrino and antineutrino flavor transition probabilities on ε_{13} and δ appear to be the same. We find that the transitions from muon to electron flavor and electron to muon flavor behave the same, where the probability ranges from about 0.3 to 0.6. The transitions from electron to tau flavor and tau to electron flavor also behave identically, ranging from 0.3 to 0.6 as well, but with maximum and minimum values reversed from those of the muon-electron flavor transitions. For transitions from muon to tau flavor, tau to muon flavor, and tau to tau flavor, the probabilities span a smaller range over the possible values for the CP-violating phase. Muon to muon flavor transition probabilities range from about 0 to 0.3 over the range of values for δ, which could be of interest considering the prediction of zero muon flavor neutrinos surviving at particular energies and δ values. Electron to electron flavor transitions, on the other hand, seem to be independent of the CP-violating phase. One example is plotted in Figure 2, again, in which L/E = 61,840 km/GeV.

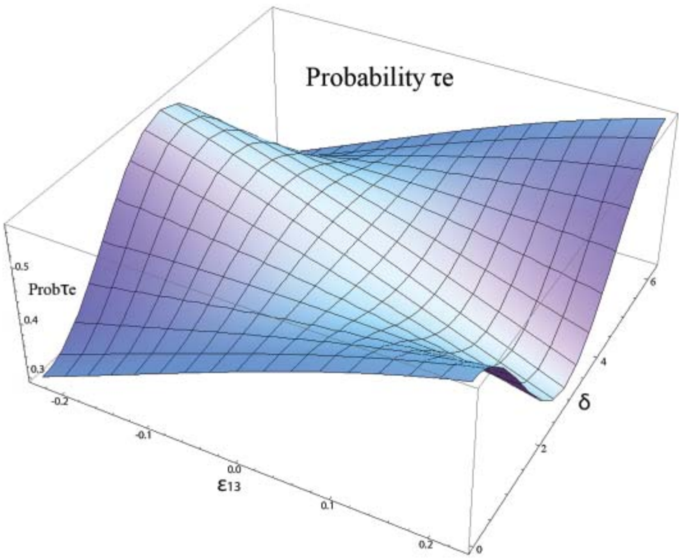

Figure 2

Probability of tau to electron flavor transition with L/E =61,840 km/GeV, ε_{12}=0, and ε_{23}=0.

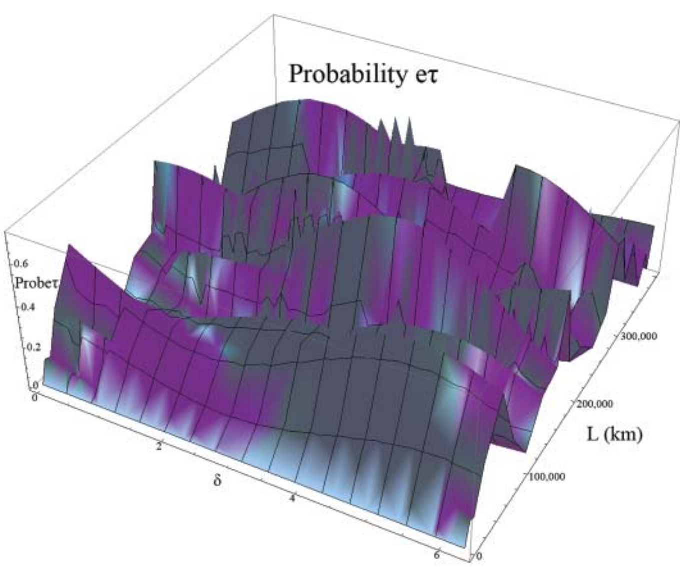

Figure 3

Probability of electron to tau flavor transition with E=1 GeV.

Using a 3-dimensional plot of each probability over δ and L/E, with E = 1 GeV, shown in Figure 3, for example, we observe that probabilities that seemed to share similar dependence on δ are not the same after all. This suggests that some behavior is lost in the previous method of analysis. We further explore the effects of the CP-violating phase on transition probabilities by choosing ε_{13} and δ to maximize CP effects, fixing L at 10,000 km and plotting over E. Here, it appears that tau to electron flavor, plotted in Figure 4, and electron to muon flavor transitions are most sensitive to CP-violation. We observe the reversed transition probabilities of antineutrinos to be sensitive to CP-violation. For example, the probability of a muon flavor antineutrino becoming an electron flavor antineutrino is sensitive to CP-violation. These plots also indicate the range of L/E for which the transition probabilities are most sensitive to CP-violation, which could be useful in designing experiments to measure CP-violation in the neutrino sector. As we see in Figure 4, this useful L/E range appears to be from about 10,000 to 30,000 km/GeV, so we are most likely to measure CP-violation at relatively low energies. Thus, experiments aimed at observing CP-violation in the neutrino sector should focus on the tau to electron and electron to muon neutrino flavor channels, as lower energies yield the best results.

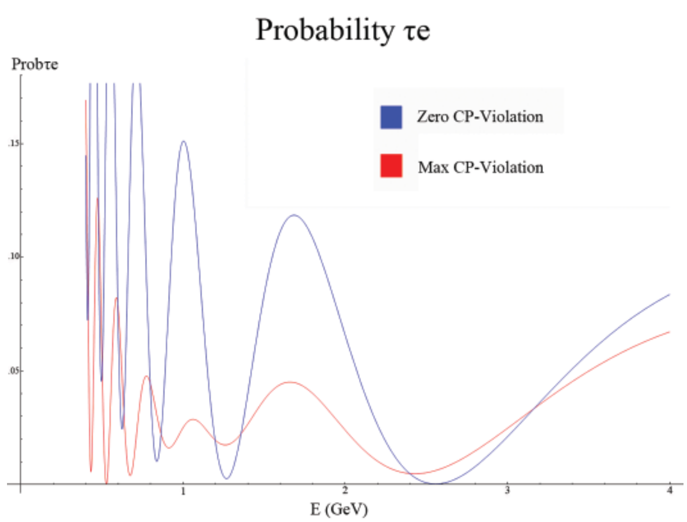

Figure 4

Probability of tau to electron flavor transition with L=10,000 km; "Zero CP-Violation" indicates a plot generated using δ=0, where "Max CP-Violation" indicates a plot generated using δ=π/2.

Channels Most Sensitive to Mass Ordering

Extending our examination of the effects of CP-violation on transition probabilities, we can explore the effects of the mass hierarchy chosen by changing the values used for the mass splittings with at least small deviations from the Tri-bimaximal values introduced, as the imaginary terms

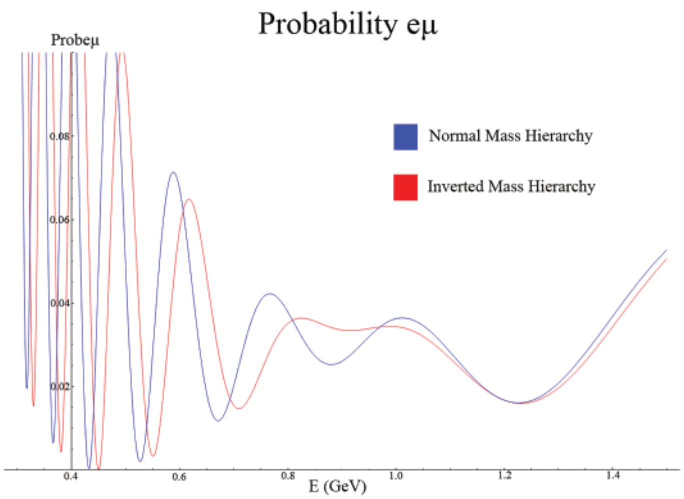

Figure 5

Probability of electron to muon flavor transition with L=10,000 km; Cases of both normal and inverted mass hierarchies are plotted.

do not appear without CP-violation. We find that the probability of transitioning from electron to muon flavor is most significantly affected by such a change, though tau to electron flavor transition probabilities are also altered. In the electron to muon flavor channel, significant effects are observed on the L/E range from 8,500 to 20,000 km/GeV. These guidelines may be used in designing experiments to identify the correct mass hierarchy.

Leptogenesis

As previously explained, our interest in CP-violation in the neutrino sector arose out of the possibility of leptogenesis as an explanation for the observed matter-antimatter asymmetry. The three conditions for the generation of baryon asymmetry are baryon number violation, charge (C) and charge-parity (CP) violation, and departure from thermal equilibrium, all of which can be satisfied under the Standard Model of particle physics (Nir, 2007). However, the effects of CP-violation in the baryogenesis formula are suppressed by small quark mixing angles and cannot produce baryon asymmetry with a magnitude comparable to the observed asymmetry. Furthermore, although baryon and lepton numbers can be violated under the standard model, the difference between them remains invariant, which forbids Majorana masses for neutrinos. It has been shown that the introduction of sterile singlet neutrinos, which are used to generate light neutrino masses, fulfills the requirements for the generation of baryon asymmetry and inevitably leads to leptogenesis (Fukugita, 1990, Luty, 1992, and Nir, 2007). The lepton asymmetry then gets converted into baryon asymmetry. We can thus calculate lepton asymmetry in terms of the neutrino and lepton mixing parameters for a particular model in hopes of providing a sufficient amount of the observed matter-antimatter asymmetry.

Below, I will investigate how leptogenesis depends on the CP-violating phase δ in a model derived from the group theory of both T', a variant of the tetrahedral group A4, and SU(5). (The tetrahedral group describes the rotational symmetry associated with a tetrahedron, a perfect geometric solid with 4 faces and 4 vertices.) Here, the T' is a family symmetry and SU(5) is the grand unification symmetry. In this model, we find that the neutrino mixing parameters depend on $\theta_c \approx 0.224$, the Cabibbo angle for quark mixing, and thus have definite values, except for ε_{12}, which can be written in terms of the CP-violating phase. This investigation may give us an idea of what magnitudes of asymmetry may be generated by leptogenesis and what values for the CP-violating phase may be desirable.

Generating Masses

Four fundamental forces have been observed in modern physics—electromagnetic, gravitational, strong, and weak—that behave differently and are mediated by force carriers of different masses. The electromagnetic force, for example, is mediated by photons. What is the reason for this number of forces? Why should they behave differently? Grand Unification Theory suggests that the weak, electromagnetic and strong forces can be unified at extremely high energies. Electroweak unification is the unification of the electromagnetic and weak forces, which are mediated by particles of very different masses and behave differently at current energy levels. This requires a spontaneous symmetry breaking as the universe cools, which is provided by the Higgs boson. It is this symmetry breaking through the Higgs interaction that gives rise to different fermion masses in the Standard Model (Higgs, 1964).

Neutrino masses, on the other hand, are not predicted by the Standard Model, which assumes that only left-handed neutrinos and right-handed antineutrinos exist, leaving neutrinos with neither Majorana nor Dirac mass terms. Even supposing that the Standard Model did give rise to neutrino masses, we still could not explain why neutrino masses are so much smaller than other fermions. By introducing sterile right-handed neutrino and left-handed antineutrino singlets, which cannot feel the weak, electromagnetic, or strong forces, we introduce both Majorana and Dirac mass terms. By examining the eigenvalues of our new neutrino mass matrix, we find that if these new sterile neutrinos are sufficiently heavy, the observed physical neutrinos will be sufficiently light. This mechanism for generating neutrino masses of the proper size is known as the see-saw mechanism (Gell-Man, 1979), because as the sterile neutrino masses increase, the physical neutrino masses decrease.

T' Models and the Usual See-Saw Realization

We now turn our attention to theoretical models, derived from the double tetrahedral group, T', in which the electroweak symmetry breaking generates the following effective neutrino mass matrix, where υ is the Standard Model Higgs Vacuum Expectation Value (VEV), Λ_L is the cutoff scale for the T' symmetry, and α_s and α_0 are the two parameters that determine the neutrino masses (Chen, 2007).

$$M_{eff}^{\nu} = \begin{bmatrix} 2\alpha_s + \alpha_0 & -\alpha_s & -\alpha_s \\ -\alpha_s & 2\alpha_s & -\alpha_s + \alpha_0 \\ -\alpha_s & -\alpha_s + \alpha_0 & 2\alpha_s \end{bmatrix} \frac{\upsilon^2}{\Lambda_L} \tag{5}$$

The above mass matrix is form-diagonalizable, meaning that it can be diagonalized by the same matrix, independent of its parameters α_s and α_0. The diagonalizing matrix, which is fixed by group theory, happens to be the Tri-bimaximal mixing matrix. This suggests that the familiar Tri-bimaximal mixing, previously just a guess that seemed to fit the available experimental data quite well, can actually be derived from group theory, yielding mass eigenvalues that depend upon just two parameters.

In the usual see-saw realization in this model, the Majorana mass matrix M_{RR}, which couples right-handed neutrinos, N, to right-handed neutrinos, and the Dirac mass matrix M_D, which couples right-handed neutrinos to left-handed neutrinos, L, are given by the interactions in Equations 6 and 7, in which H is the Higgs boson and y is a coupling constant (Chen and King, 2009).

$$M_{RR} = \overline{N^c} N((\varphi_s) + (u)) = \begin{bmatrix} 2\alpha_s + \alpha_0 & -\alpha_s & -\alpha_s \\ -\alpha_s & 2\alpha_s & -\alpha_s + \alpha_0 \\ -\alpha_s & -\alpha_s + \alpha_0 & 2\alpha_s \end{bmatrix} \Lambda \tag{6}$$

$$M_D = yH\overline{L}N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} y\upsilon \tag{7}$$

The effective mass matrix for neutrinos is given by

$$M_{\nu}^{eff} = M_D M_{RR}^{-1} M_D^T = U_{TBM}^T \, diag(m_1, m_2, m_3) U_{TBM} \tag{8}$$

which has three light neutrino mass eigenvalues m_1, m_2, and m_3.

$$diag(m_1, m_2, m_3) = diag\left(\frac{1}{3\alpha_s + \alpha_0}, \frac{1}{\alpha_0}, \frac{1}{3\alpha_s - \alpha_0}\right) \frac{y^2 \upsilon^2}{\Lambda} \tag{9}$$

M_{RR} is diagonalized by the Tri-bimaximal mixing matrix, as we expect from its similarities in form to the mass matrix derived from group theory, as discussed earlier in Equation 5.

$$M_{RR}^{diag} = U_{TBM}^T M_{RR} U_{TBM} = diag(3\alpha_s + \alpha_0, \alpha_0, 3\alpha_s - \alpha_0)\Lambda \tag{10}$$

When we rotate M_D to the basis in which M_{RR} is diagonal, which we will need for our leptogenesis calculations, we find

$$M_D^{'} = M_D U_{TBM} = y\upsilon \begin{bmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{11}$$

Determining the Mixing Parameters

From the interactions of the left-handed neutrinos with charged leptons mediated by the W-gauge boson, we find that the neutrino mixing matrix $U_{MNS} = U_{eL} U_{\nu}^{\dagger}$, where U_{eL} is the diagonalization matrix of the charged lepton mixing matrix, parameterized in the model considered by $\theta_c \approx 0.224$, the Cabibbo mixing angle for quarks, and $U_{\nu}^{\dagger}$ is the Tri-bimaximal mixing matrix.

$$U_{eL} = \begin{bmatrix} 1 & \frac{1}{3}\theta_c e^{-i\delta} & 0 \\ -\frac{1}{3}\theta_c e^{i\delta} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$U_{MNS} = \begin{bmatrix} \frac{-6 + e^{-i\delta}\theta_c}{3\sqrt{6}} & \frac{3 + e^{-i\delta}\theta_c}{3\sqrt{3}} & \frac{e^{-i\delta}\theta_c}{3\sqrt{2}} \\ \frac{3 + 2e^{i\delta}\theta_c}{3\sqrt{6}} & \frac{-3 + e^{i\delta}\theta_c}{3\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \tag{13}$$

In this model, δ is again the only source of CP-violation in our neutrino mixing matrix, where other CP-violating phases have vanished (Chen and Mahanthappa, 2009), but this may not be the same δ that we discussed in the first half of this paper. To determine the relationship between the two CP-violating phases, we calculate the Jarlskog CP-invariant, which is the term that governs flavor transition probabilities and is invariant across different parameterizations of the neutrino mixing matrix. We find in the cases of both mixing matrices that the Jarlskog CP-invariant is proportional to sin(δ); thus, we conclude that we are dealing with the same CP-violating phase delta as before.

By comparing this new form of UMNS to the general neutrino mixing matrix (before expansion) in terms of the neutrino mixing parameters, θ_{23}, θ_{13}, θ_{12}, and δ, we can find the deviations from the Tri-bimaximal mixing angles in terms of θ_c by mapping corresponding elements of the matrices to each other and isolating each parameter. Performing these calculations gives us the neutrino mixing angles in terms of the quark mixing angle, unifying quark and neutrino mixing under the same parameter, our goal in a model derived from Grand Unification Theory (Chen, 2007). We find ε_{13}=arcsin($3\theta_c/\sqrt{2}$), $\varepsilon_{12}\approx$ -arctan($1/\sqrt{2}$) +arctan($\sqrt{(1-\cos(\delta)\,\theta_c)/\sqrt{2}}$), and ε_{23}= 0. Here, the CP-violating phase δ is still allowed to vary, but we find that our approximations are best when δ is equal to an integer multiple of $\pi/2$, i.e. maximum CP-violation, yielding $\varepsilon_{13}\approx 0.0528219$, $\varepsilon_{12}\approx 0$, $\varepsilon_{23}= 0$, and δ=$\pi/2$ or $3\pi/2$ in this particular model. Plots of flavor transition probabilities with these values for the mixing angles are similar to those generated with the Tri-bimaximal mixing values and are therefore reasonable.

At this point, we have obtained all of the information that we need to calculate leptogenesis in terms of the neutrino mixing parameters explored in the first half of this paper.

Calculating the Matter-Antimatter Asymmetry

To calculate leptogenesis, we use Equation 14, in which $\Gamma(N_1\to\ell_\alpha H)$ is the width of the decay of the lightest right-handed neutrino into a lepton of flavor α and a Higgs boson (Chen, 2008).

$$\varepsilon_1=\frac{\sum_\alpha[\Gamma(N_1\to\ell_\alpha H)-\Gamma(N_1\to\bar{\ell}_\alpha\bar{H})]}{\sum_\alpha[\Gamma(N_1\to\ell_\alpha H)+\Gamma(N_1\to\bar{\ell}_\alpha\bar{H})]}\approx\frac{1}{8\pi}\frac{1}{(h_\nu h_\nu^+)_{11}}\sum_{i=2,3}\mathrm{Im}\left\{(h_\nu h_\nu^+)_{1i}^2\right\}\left[f\left(\frac{M_i^2}{M_1^2}\right)+g\left(\frac{M_i^2}{M_1^2}\right)\right]$$
$$f(x)=\sqrt{x}\left[1-(1+x)\ln\left(\frac{1+x}{x}\right)\right],\quad g(x)=\frac{\sqrt{x}}{1-x}\qquad(14)$$

We use the rotated matrices from equations 10, 11 and 12, where M is the diagonalized M_{RR} and h_ν, given in Equation 15 is the product of U_{eL} and M_D', where M_D' is the rotated M_D from above, y is a coupling constant, and υ is the Higgs VEV.

$$h_\nu=\begin{bmatrix}-\sqrt{\frac{2}{3}}\upsilon y+\frac{\upsilon y e^{-i\delta}\theta_c}{3\sqrt{6}} & \frac{\upsilon y}{\sqrt{3}}+\frac{\upsilon y e^{-i\delta}\theta_c}{3\sqrt{3}} & \frac{\upsilon y e^{-i\delta}\theta_c}{3\sqrt{2}}\\ \frac{\upsilon y}{\sqrt{6}}+\frac{\sqrt{2}\upsilon y e^{i\delta}\theta_c}{3\sqrt{3}} & \frac{\upsilon y}{\sqrt{3}}-\frac{\upsilon y e^{i\delta}\theta_c}{3\sqrt{3}} & \frac{\upsilon y}{\sqrt{2}}\\ \frac{\upsilon y}{\sqrt{6}} & \frac{\upsilon y}{\sqrt{3}} & -\frac{\upsilon y}{\sqrt{2}}\end{bmatrix}\qquad(15)$$

We find that $h_\nu h_\nu^\dagger$ is diagonal in this model. This indicates that the lepton-antilepton asymmetry in this model will be zero. We consider, however, the possibility of significant flavor effects in this calculation, which are relevant if leptogenesis takes place below 10^{12} GeV.

To take flavor effects into account, we use Equation 16, which is summed over the three flavors, denoted by α. In this formula, M_1 is the lightest heavy, sterile, right-handed neutrino mass eigenvalue, m_β and m_ρ are the light physical neutrino mass eigenvalues, and ρ and β run from 1 to 3. R is a matrix given by the product of h_ν, U_{MNS}, the two diagonal mass matrices for light and heavy neutrinos, and the Higgs VEV.

$$\varepsilon_\alpha=-\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}(\sum_{\beta\rho}m_\beta^{1/2}m_\rho^{3/2}U_{\alpha\beta}^*U_{\alpha\rho}R_{1\beta}R_{1\rho})}{\sum_\beta m_\beta\left|R_{1\beta}\right|^2}\qquad(16)$$

This equation does not simplify easily because the complicated coefficients of the various terms depend differently on these mass eigenvalues. From this expression, we see that the asymmetry is proportional to some combination of trigonometric functions of the CP-violating phase, which we might expect as it is a complex phase introduced to the mixing matrix. By setting θ_c= 0.224, M_1=10^{10} GeV, υ= 174

GeV, $m_1 = 0.0156$ eV, $m_2 = 0.0179$ eV, $m_3 = 0.0514$ eV, and $y = 1$, where the Higgs VEV and the Cabibbo angle are set to their accepted values, the coupling constant y has little effect, and the light left-handed neutrino mass eigenvalues are predicted by the model, we can plot the lepton asymmetry over the allowed range of the CP-violating phase δ, as shown in Figure 6. We find that the asymmetry appears to depend on –sin(δ), with a maximum value of about 4.25×10^{-6} at $\delta=3\pi/2$. This suggests that the value of the CP-violating phase may be $3\pi/2$, rather than the expected $\pi/2$, as $3\pi/2$ generates the maximum asymmetry between leptons and antileptons. When we plot the above flavor transition probabilities, we find that such a change in δ switches the neutrino transition probabilities with those of the anti-neutrinos, as we might expect given the switch in the sign, providing little additional information. On the other hand, when we reverse the mass hierarchy, we find that the asymmetry is rescaled to about one third of that for the case of the normal mass hierarchy. This suggests that the normal mass hierarchy is advantageous because we want to generate an asymmetry large enough to explain the observed matter-antimatter asymmetry.

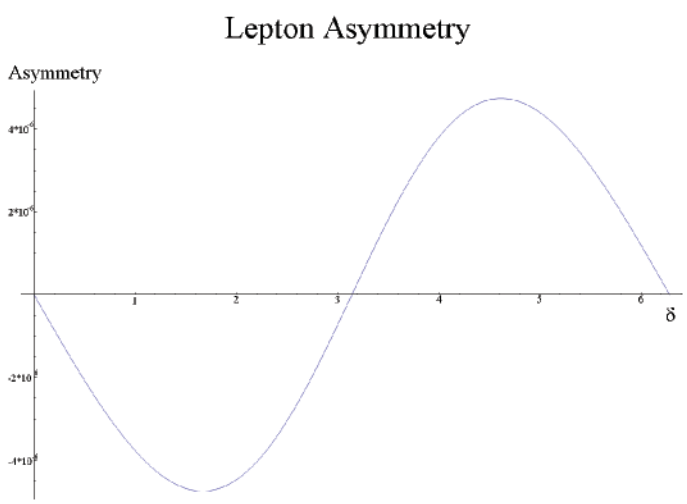

Figure 6

Asymmetry between leptons and antileptons in chosen T' model.

From Figure 6, we can conclude that leptogenesis is a possible mechanism for generating the observed asymmetry between matter and antimatter, supporting a possible explanation for one of the greatest mysteries in modern physics. We can also gather information that is useful in predicting values for the CP-violating phase and determining the mass hierarchy. In the future, I will attempt to calculate the asymmetry plotted above in terms of the neutrino mixing angles. This may provide further insight into the possibility of leptogenesis and the values of the neutrino mixing angles.

Conclusion

By calculating transition probabilities in terms of the neutrino mixing parameters and varying the values of the mixing angles and phase over the experimentally allowed range, I have identified the electron to muon and tau to electron flavor channels as the most sensitive to CP-violation and changes in the mass ordering. I have also determined the most sensitive distance to energy ratios at which we may hope to observe CP-violation.

If CP-violation is observed, and possibly measured, in the neutrino sector, leptogenesis is a possible means of explaining the observed asymmetry between matter and antimatter. I have calculated the dependence of the asymmetry between leptons and antileptons on the CP-violating phase for a particular model derived from the T' group, which I have plotted above. This dependence suggests that the maximum asymmetry between matter and antimatter occurs when the CP-violating phase is equal to $3\pi/2$ and the mass hierarchy for neutrinos is normal. These results directly give us the relationship between CP-violation in the neutrino sector and the lepton-antilepton asymmetry, which we may use to calculate the asymmetry between baryons and anti-baryons in hopes of reaching and explaining the observed asymmetry.

My exploration of transition probabilities and leptogenesis in terms of the neutrino mixing parameters will serve as a guide for the design of future neutrino experiments and neutrino mixing models, improving our understanding of the neutrino sector of particle physics. As we improve the values of our experimental parameters, possibly even finding values for the light and heavy neutrino mass eigenvalues and the CP-violating phase in the neutrino sector at some point in the future, we can explore the lepton-antilepton asymmetry for various models by performing calculations similar to those in this paper. Hopefully this will lead to finding a model that matches experimental data and explains one of the great mysteries of our time.

Acknowledgements

I would like to thank my mentor Dr. Mu-Chun Chen for all of her help in the research process. Her never-ending patience and support was invaluable throughout this process. I would also like to thank Dr. Haibo Yu for his council in Dr. Chen's absence, Dr. Gunar Schirner for his aid in preparing to present my thesis, and Shaun Wilson for his editing and formatting expertise.

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